



Lecture 4: Feature Model and Code Analysis

Gregory Gay TDA 594/DIT 593 - November 11, 2021





Propositional Logic

- Mandatory: If parent is selected, the child must be.
 - mandatory(p, f) \equiv f \Leftrightarrow p
- **Optional:** Child may only be chosen if the parent is.
 - optional(p, f) \equiv f \Rightarrow p



Propositional Logic

- Alternative: Choose exactly one
 - alternative(p, {f₁,...,f_n}) \equiv ((f₁ V ... V f_n) \Leftrightarrow p) $\wedge_{(fi,fj)} \neg (f_i \wedge f_j)$
- Or: Choose at least one

• or(p, {f₁,...,f_n})
$$\equiv$$

((f₁ \lor ... \lor f_n) \Leftrightarrow p)









Analyses of Feature Models

- Is a feature selection valid?
- Is the feature model consistent?
- Do our assumptions hold (testing)?
- Which features are mandatory?
- Which features can never be selected (dead)?
- How many valid selections does model have?
- Are two models equivalent?
- Given partial selection, what must be included?
- What selections give best cost/size/performance?





Valid Feature Selection

- Translate model into a propositional formula φ.
- Assign true to each selected feature, false to rest.
- Assess whether φ is true.
 - If yes, valid selection.







Example - Graph Library



- $\phi = \texttt{GraphLibrary} \land \texttt{EdgeType} \land (\texttt{Directed} \lor \texttt{Undirected}) \land \neg(\texttt{Directed} \land \texttt{Undirected}) \land ((\texttt{Cycle} \lor \texttt{ShortestPath} \lor \texttt{MST}) \Leftrightarrow \texttt{Algorithm}) \land (\texttt{Cycle} \Rightarrow \texttt{Directed})$
 - $\land ((\texttt{Prim} \lor \texttt{Kruskal}) \Leftrightarrow \texttt{MST}) \land \neg (\texttt{Prim} \land \texttt{Kruskal}) \land (\texttt{MST} \Rightarrow (\texttt{Undirected} \land \texttt{Weighted}))$

.



 $(T) \land \neg (F) \land (T)$



Example - Graph Library



Selection: {GraphLibrary, EdgeType, Directed}

$$\varphi = T \land T \land (T \lor F) \land \neg (T \land F)$$

$$\land ((F \lor F \lor F) \Leftrightarrow F) \land (F \Rightarrow F)$$

$$\land ((F \lor F) \Leftrightarrow F) \land \neg (F \land F) \land (F \Rightarrow (F \land F))$$

$$\varphi = T \land T \land (T) \land \neg (F)$$



 $\phi = \texttt{GraphLibrary} \land \texttt{EdgeType} \land (\texttt{Directed} \lor \texttt{Undirected}) \land \neg(\texttt{Directed} \land \texttt{Undirected})$

 $\land ((\texttt{Cycle} \lor \texttt{ShortestPath} \lor \texttt{MST}) \Leftrightarrow \texttt{Algorithm}) \land (\texttt{Cycle} \Rightarrow \texttt{Directed})$

 $\land ((\texttt{Prim} \lor \texttt{Kruskal}) \Leftrightarrow \texttt{MST}) \land \neg (\texttt{Prim} \land \texttt{Kruskal}) \land (\texttt{MST} \Rightarrow (\texttt{Undirected} \land \texttt{Weighted}))$

Prim

Kruskal





Consistent Feature Models

- A consistent model has 1+ valid selections.
 - Inconsistent models do not have any valid selection.
- Contradictory constraints are common.
- Find feature selection that results in $\varphi = true$
 - NP-complete problem, but SAT solvers can often find solutions quickly.



Boolean Satisfiability (SAT)

- Find assignments to Boolean variables $X_1, X_2, ..., X_n$ that results in expression ϕ evaluating to true.
- Defined over expressions written in **conjunctive normal form**.

•
$$\varphi = (X_1 \lor \neg X_2) \land (\neg X_1 \lor X_2)$$

- $(X_1 \vee \neg X_2)$ is a **clause**, made of variables, \neg , \vee
- Clauses are joined with $\boldsymbol{\wedge}$





Conjunctive Normal Form

- Variables: X_1, X_2, X_3, X_4, X_5
- Clauses (using only \vee (or) and \neg (not)):
 - $(\neg X_2 \lor X_5), (X_1 \lor \neg X_3 \lor X_4), (X_4 \lor \neg X_5), (X_1 \lor X_2)$
- Expression ϕ joins clauses with Λ (and)
 - $(\neg X_2 \lor X_5) \land (X_1 \lor \neg X_3 \lor X_4) \land (X_4 \lor \neg X_5) \land (X_1 \lor X_2)$

Boolean Satisfiability

- Find assignment to X_1, X_2, X_3, X_4, X_5 to solve • $(\neg X_2 \lor X_5) \land (X_1 \lor \neg X_3 \lor X_4) \land (X_4 \lor \neg X_5) \land (X_1 \lor X_5)$
- One solution: 1, 0, 1, 1, 1
 - $(\neg X_2 \lor X_5) \land (X_1 \lor \neg X_3 \lor X_4) \land (X_4 \lor \neg X_5) \land (X_1 \lor X_2)$
 - $(\neg 0 \lor 1) \land (1 \lor \neg 1 \lor 1) \land (1 \lor \neg 1) \land (1 \lor 0)$
 - (1) \wedge (1) \wedge (1) \wedge (1)

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Transformation Rules

• De Morgan's Laws

•
$$\neg(X \lor Y) \equiv \neg X \land \neg Y$$

•
$$\neg(X \land Y) \equiv \neg X \lor \neg Y$$

• Distributivity

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- $X \lor (Y \land Z) \equiv (X \lor Y) \land (X \lor Z)$
- $X \land (Y \lor Z) \equiv (X \land Y) \lor (X \land Z)$
- Double Negation





Transformation Rules

- X⇔Y
 - X is equivalent to Y

•
$$\equiv (X \Rightarrow Y) \land (Y \Rightarrow X)$$

- $(X \Rightarrow Y) \equiv (\neg X \lor Y)$
- If X is true, Y is also true.
- If X is false, Y can be either true or false.
- $\equiv (\neg X \lor Y) \land (\neg Y \lor X)$





Transformation into CNF



VOD ∧ (VOD ⇔ (Record ∨ Play)) ∧ (Mobile ⇔ Play ∧ \neg TV) ∧ (TV ⇔ Play ∧ \neg Mobile)



Transformation into CNF

- VOD ∧ (VOD ⇔ (Record ∨ Play)) ∧ (Mobile ⇔ (Play ∧ ¬TV)) ∧ (TV ⇔ (Play ∧ ¬Mobile))
 - (VOD ⇔ (Record ∨ Play))
 - \equiv (VOD \Rightarrow (Record \lor Play)) \land ((Record \lor Play) \Rightarrow VOD)
 - (¬VOD V (Record V Play)) ∧ (¬(Record V Play) V VOD)
 - (¬VOD V (Record V Play)) ∧ (¬Record V VOD) ∧ (¬Play V VOD)
 - (Mobile \Leftrightarrow (Play $\land \neg TV$))
 - (Mobile ∨ TV ∨ ¬Play) ∧ (¬Mobile ∨ Play) ∧ (¬Mobile ∨ ¬TV)
 - (TV \Leftrightarrow (Play $\land \neg$ Mobile))
 - $\equiv (TV \lor Mobile \lor \neg Play) \land (\neg TV \lor Play) \land (\neg TV \lor \neg Mobile)$

DIMACS Format



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- Map feature names to integer IDs.
 - VOD = 1
 - Record = 2
 - Play = 3
 - TV = 4
 - Mobile = 5

DIMACS Format



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VOD 🖊

(¬VOD ∨ (Record ∨ Play)) ∧ (¬Record ∨ VOD) ∧ (¬Play ∨ VOD)

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(Mobile \lor TV \lor \neg Play) \land (\neg Mobile \lor Play) \land (\neg Mobile \lor \neg TV) \land

 $(TV \lor Mobile \lor \neg Play) \land (\neg TV \lor Play) \land (\neg TV \lor \neg Mobile)$

1 \Lambda

 $(\neg 1 \lor (2 \lor 3)) \land (\neg 2 \lor 1) \land (\neg 3 \lor 1) \land$ $(5 \lor 4 \lor \neg 3) \land (\neg 5 \lor 3) \land (\neg 5 \lor \neg 4) \land$ $(4 \lor 5 \lor \neg 3) \land (\neg 4 \lor 3) \land (\neg 4 \lor \neg 5)$











| | | omments |
|------------|--|---------|
| 1 | Add DIMACs header P_Cr | nf 5 10 |
| -1 V 2 V 3 | • Comments | |
| -2 V 1 | Indicates CNF format Number of uprichlas | 23 |
| -3 V 1 | Number of variables Number of CNF clauses -2 1 | |
| 5 V 4 V -3 | -3 1 | |
| -5 V 3 | 54 | -3 |
| -5 V -4 | -5 3 | |
| 4 ∨ 5 ∨ -3 | -5 - | 4 |
| -4 \V 3 | 4 5 | -3 |
| -4 V -5 | -4 3 | |
| | -4-5 | 5 |



Using a SAT Solver

- Identify assignment that results in true outcome.
 - VOD ∧ (¬VOD ∨ (Record ∨ Play)) ∧ (¬Record ∨ VOD) ∧ (¬Play ∨ VOD) ∧ (Mobile ∨ TV ∨ ¬Play) ∧ (¬Mobile ∨ Play) ∧ (¬Mobile ∨ Play) ∧ (¬TV) ∧ (TV ∨ Mobile ∨ ¬Play) ∧ (¬TV ∨ Play) ∧ (¬TV ∨ ¬Mobile)
 - A satisfying assignment: (1, 1, 1, 1, 0)
- Returns satisfying assignment.
 - May return all satisfying assignments found.
 - If not satisfiable, may offer information on why.

https://bit.ly/3BVwMZc

F



Activity

- Start with A/B.
 Do C/D if time.
- Translate model into propositional logic formula.
- Provide two valid and two invalid features.
- Is it consistent? If not, why not?



D

F

G

С

В

E



D

G



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- Translate model into propositional logic formula.
- Provide two valid and two invalid features.
- Is it consistent? If not, why not?



 $\begin{array}{l} \mathsf{A} \land (\mathsf{B} \Rightarrow \mathsf{A}) \land (\mathsf{C} \Leftrightarrow \mathsf{A}) \land (\mathsf{D} \Rightarrow \mathsf{A}) \land \\ ((\mathsf{C} \Leftrightarrow (\mathsf{E} \lor \mathsf{F})) \land \neg (\mathsf{E} \land \mathsf{F})) \land ((\mathsf{E} \lor \mathsf{F}) \Rightarrow \mathsf{D})) \end{array}$

- Valid: A, B, C, D, F ; A, C, D, E
- Invalid: A, B, C, D, E, F ; A, B, C, E
- Is it consistent: Yes

Solution (B)

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- Translate model into propositional logic formula.
- Provide two valid and two invalid features.
- Is it consistent? If not, why not?



 $A \land (B \Leftrightarrow A) \land (C \Rightarrow A) \land (D \Rightarrow A) \land$ $((C \Leftrightarrow (E \lor F)) \land \neg (E \land F)) \land (G \Rightarrow D) \land (D \Rightarrow \neg B)$ \land $(E \Rightarrow G)$

- Valid: A, B ; A, B, C, F
- Invalid: A, B, D, G ; A, B, C, E
- It is consistent: Yes, but D, E, and G are dead features (because B is mandatory).

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Solution (C)

- Translate model into propositional logic formula.
- Provide two valid and two invalid features.
- Is it consistent? If not, why not?

 $A \bigwedge ((B \lor C \lor D) \Leftrightarrow A) \bigwedge (E \Leftrightarrow B) \bigwedge (F \Rightarrow D) \bigwedge (G \Rightarrow D)$

- Valid: A, C ; A, B, C, D, E, F, G
- Invalid: A, B, C; A, C, E
- It is consistent: Yes (just remember that B and E need to come as a pair)



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Solution (D)

- Translate model into propositional logic formula.
- Provide two valid and two invalid features.
- Is it consistent? If not, why not?

 $\begin{array}{l} A \land (B \Rightarrow A) \land (C \Leftrightarrow A) \land (D \Leftrightarrow B) \land (E \Rightarrow C) \land (F \Rightarrow C) \land \\ (F \Rightarrow E) \land (D \Leftrightarrow E) \end{array}$

- Valid: A, C ; A, B, C, D, E
- Invalid: A, B, C, D ; A, C, F
- It is consistent: Yes, but remember that if you have F, you need E, D, and B as well.







SAT Solver Process

- Express in conjunctive normal form:
 - $\varphi = (\neg x2 \lor x5) \land (x1 \lor \neg x3 \lor x4) \land (x4 \lor \neg x5) \land (x1 \lor x2)$
- Choose assignment based on how it affects each clause it appears in.
 - What happens if we assign x2 = true?
 - If any clauses now false, don't apply that value.
 - Continue until CNF expression is satisfied.





Branch & Bound Algorithm

- Set variable to true or false.
- Apply that value.
- Does value satisfy the clauses that it appears in?
 - If so, assign a value to the next variable.
 - If not, backtrack (bound) and apply the other value.
- Prunes branches of the boolean decision tree as values are applied.





Branch & Bound Algorithm

 $\varphi = (\neg x2 \lor x5) \land (x1 \lor \neg x3 \lor x4) \land (x4 \lor \neg x5) \land (x1 \lor x2)$

- Set x1 to false. φ = (¬x2 ∨ x5) ∧ (0 ∨ ¬x3 ∨ x4) ∧ (x4 ∨ ¬x5) ∧ (0 ∨ x2)
- 2. Set x2 to false.

 $\varphi = (\mathbf{1} \lor x5) \land (\mathbf{0} \lor \neg x3 \lor x4) \land (x4 \lor \neg x5) \land (\mathbf{0} \lor \mathbf{0})$

3. Backtrack and set x2 to true.

 $\varphi = (\mathbf{0} \lor x5) \land (\mathbf{0} \lor \neg x3 \lor x4) \land (x4 \lor \neg x5) \land (\mathbf{0} \lor \mathbf{1})$

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DPLL Algorithm

- Set a variable to true/false.
 - Apply that value to the expression.
 - Remove all satisfied clauses.
 - If assignment does not satisfy a clause, then remove that variable from that clause.
 - If this leaves any **unit clauses** (single variable clauses), assign a value that removes those next.
- Repeat until a solution is found.



DPLL Algorithm

 $\varphi = (\neg x2 \lor x5) \land (x1 \lor \neg x3 \lor x4) \land (x4 \lor \neg x5) \land (x1 \lor x2)$

1. Set x2 to false.

 $\varphi = (\neg \mathbf{0} \lor x5) \land (x1 \lor \neg x3 \lor x4) \land (x4 \lor \neg x5) \land (x1 \lor \mathbf{0})$ $\varphi = (x1 \lor \neg x3 \lor x4) \land (x4 \lor \neg x5) \land (x1)$

2. Set x1 to true.

$$\varphi = (\mathbf{1} \lor \neg \mathbf{x}3 \lor \mathbf{x}4) \land (\mathbf{x}4 \lor \neg \mathbf{x}5) \land (\mathbf{1})$$

$$\varphi = (\mathbf{x}4 \lor \neg \mathbf{x}5)$$

3. Set x4 to false, then x5 to false.

$$\varphi = (\mathbf{0} \lor \neg x5)$$

$$\varphi = (\neg \mathbf{0})$$





Let's take a break!

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Testing Facts About the Model





Testing Facts about Models

- A fact that should be true encoded as formula ψ .
- Check whether $\phi \land \neg \psi$ is satisfiable.
 - Is there a valid feature selection for ϕ that does not satisfy constraint $\psi?$
 - If yes, there is a problem with the model.



Example - Graph Library



 $\phi = \texttt{GraphLibrary} \land \texttt{EdgeType} \land (\texttt{Directed} \lor \texttt{Undirected}) \land \neg(\texttt{Directed} \land \texttt{Undirected})$

 $\land ((Cycle \lor ShortestPath \lor MST) \Leftrightarrow Algorithm) \land (Cycle \Rightarrow Directed)$

 $\land ((\texttt{Prim} \lor \texttt{Kruskal}) \Leftrightarrow \texttt{MST}) \land \neg (\texttt{Prim} \land \texttt{Kruskal}) \land (\texttt{MST} \Rightarrow (\texttt{Undirected} \land \texttt{Weighted}))$





Dead and Mandatory Features

- A dead feature is never used.
- A mandatory feature is always used.
- Given model φ and feature F:
 - 1+ valid selection with F if ($\phi \land F$) is satisfiable.
 - 1+ valid selection without F if ($\phi \land \neg F$) is satisfiable.
 - Feature is dead if no selection with it $(\neg(\phi \land F))$
 - Feature is mandatory if no selection without it (¬(φ Λ ¬F))



Example - Graph Library



- No dead features.
 - If Undirected made mandatory, Directed and Cycle would be dead.
 - GraphLibrary and EdgeType are mandatory.

- $\phi = \texttt{GraphLibrary} \land \texttt{EdgeType} \land (\texttt{Directed} \lor \texttt{Undirected}) \land \neg(\texttt{Directed} \land \texttt{Undirected})$
 - $\land ((\texttt{Cycle} \lor \texttt{ShortestPath} \lor \texttt{MST}) \Leftrightarrow \texttt{Algorithm}) \land (\texttt{Cycle} \Rightarrow \texttt{Directed})$
 - $\land ((\texttt{Prim} \lor \texttt{Kruskal}) \Leftrightarrow \texttt{MST}) \land \neg (\texttt{Prim} \land \texttt{Kruskal}) \land (\texttt{MST} \Rightarrow (\texttt{Undirected} \land \texttt{Weighted}))$





Constraint Propagation

- **Constraint Propagation** hiding unavailable features after we make **partial selections**.
- Feature selection often iterative:
 - Feature selected, deselected, or no decision made.
- Partial feature selection:
 - Set of selected features (S \subseteq F)
 - Set of deselected features (D \subseteq F, with S \cap D = \emptyset)





Constraint Propagation

- Partial feature selection
 - $pfs(S,D) = \forall (s \in S) \ s \land \forall (d \in D) \neg d$
- Partial selection is valid if ($\phi \land pfs(S,D)$) satisfiable
- F deactivated if (φ ∧ pfs(S,D) ∧ F) not satisfiable.
- F activated if (φ ∧ pfs(S,D) ∧ ¬F) not satisfiable.





Example - Graph Library



 $\phi = \texttt{GraphLibrary} \land \texttt{EdgeType} \land (\texttt{Directed} \lor \texttt{Undirected}) \land \neg(\texttt{Directed} \land \texttt{Undirected})$

 $\land ((Cycle \lor ShortestPath \lor MST) \Leftrightarrow Algorithm) \land (Cycle \Rightarrow Directed)$

 $\land ((\texttt{Prim} \lor \texttt{Kruskal}) \Leftrightarrow \texttt{MST}) \land \neg (\texttt{Prim} \land \texttt{Kruskal}) \land (\texttt{MST} \Rightarrow (\texttt{Undirected} \land \texttt{Weighted}))$

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Comparing Feature Models







Comparing Feature Models

- Models are equivalent if formulae are equivalent.
 ¬(φ₁ ⇔ φ₂) is not satisfiable.
- ϕ_1 is a specialization of ϕ_2 if $(\phi_2 \Rightarrow \phi_1)$ • and ϕ_2 is a generalization of ϕ_2
 - and ϕ_2 is a generalization of ϕ_1
- SAT solver can compare two models.



Use SAT Solver

to prove

 $\phi_{right} \Leftrightarrow \phi_{left}$

Example - Graph Library



$$\begin{split} \phi_{\texttt{left}} &= \texttt{Algorithm} \land ((\texttt{Cycle} \lor \texttt{ShortestPath} \lor \texttt{MST}) \Leftrightarrow \texttt{Algorithm}) \\ \phi_{\texttt{right}} &= \texttt{Algorithm} \land (\texttt{Cycle} \Rightarrow \texttt{Algorithm}) \land (\texttt{ShortestPath} \Rightarrow \texttt{Algorithm}) \\ &\land (\texttt{MST} \Rightarrow \texttt{Algorithm}) \land (\texttt{Cycle} \lor \texttt{ShortestPath} \lor \texttt{MST}) \end{split}$$





Feature-to-Code Mappings





Feature-To-Code Mappings

- Feature models describe the problem space.
- Models are implemented in source code.
- Similar analyses can examine mapping of feature models to code.
 - Which code assets are never used?
 - Which code assets are always used?
 - Which features have no influence on product portfolio?

Dead Code

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- Features that can never be incorporated.
- Feature B, in the code, required Feature A to also be selected.
- Model states that A and B are mutually exclusive.





Presence Conditions

• Describes the set of products containing a code fragment.

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- pc(c) = (conditions for c to be included in a product)
 - pc(line 3) = A
 - pc(line 5) = A \land B
 - pc(line 8) = ¬ A



- pc(lines 3-5) = A \wedge B
- pc(lines 3-8) = $A \land B \land \neg A$
 - (cannot be included in any product)

Dead Code

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- Fragment is dead if never included in any product.
 - φ represents all valid products.
 - Fragment C is dead iff (φ Λ pc(C)) is not satisfiable.

С pc() 1 line 1 True Program #ifdef A 3 line 3 Α #ifdef B ΑΛΒ 5 line 5 6 #endif Α В 7 #else ¬Α 8 line 8 #endif 9

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 $\varphi = \operatorname{Program} \land (A \lor B) \land \neg (A \land B)$ ($\varphi \land pc(\operatorname{line} 5)$) is not satisfiable: Program $\land (A \lor B) \land \neg (A \land B) \land (A \land B)$

Mandatory Code

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- Fragment is mandatory if always included in a product.
 - φ represents all valid products.
 - Fragment C is mandatory iff (φ Λ ¬pc(C)) is not satisfiable.



$$\begin{split} \phi = & \mathsf{GraphLibrary} \land \mathsf{EdgeType} \land (\mathsf{Directed} \lor \mathsf{Undirected}) \land \neg(\mathsf{Directed} \land \mathsf{Undirected}) \\ \land ((\mathsf{Cycle} \lor \mathsf{ShortestPath} \lor \mathsf{MST}) \Leftrightarrow \mathsf{Algorithm}) \land (\mathsf{Cycle} \Rightarrow \mathsf{Directed}) \\ \land ((\mathsf{Prim} \lor \mathsf{Kruskal}) \Leftrightarrow \mathsf{MST}) \land \neg(\mathsf{Prim} \land \mathsf{Kruskal}) \land (\mathsf{MST} \Rightarrow (\mathsf{Undirected} \land \mathsf{Weighted}))) \end{split}$$

If code implemented correctly, the fragment for EdgeType will be mandatory.





We Have Learned

- Feature Models can be expressed using propositional logic formulae (φ).
 - Based on model and cross-tree constaints.
- Valid feature selections result in (ϕ = true).
- SAT Solvers can identify valid configurations.
 - If none can be found, the model is inconsistent.
 - Enables many different model analyses.





We Have Learned

- Feature-Model Analysis
 - Check properties of model are true.
 - Dead and mandatory features
 - Effects of partial selections
 - Comparisons between two models
- Mapping of models and code
 - Dead and mandatory code



Next Time

Variability Implementation

- Assignment 1
 - Due November 14
 - Reach out to supervisors (and me) with questions
- Assignment 2
 - Due November 21
 - Feature modelling and analysis for mobile robots





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