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Lecture 10: Structural Testing - Paths and Data Flow

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DIT636/DAT560 - February 18, 2026

Test Adequacy Criteria

Compromise between the impossible and the inadequate

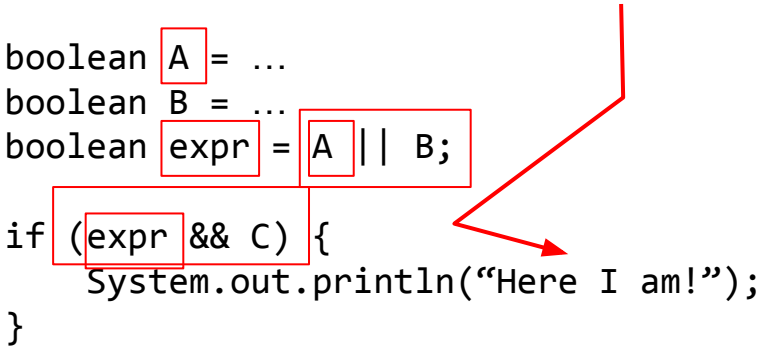
- Can we measure “good testing”?
- **Test adequacy criteria** “score” tests by measuring completion of **test obligations**.
  - Checklists of properties that must be met by test cases.

Structural Coverage Criteria

- Criteria based on exercising:
  - Statements (nodes of CFG)
  - Branches (edges of CFG)
  - Decisions and Conditions
  - Paths
  - … and many more
- Measurements used as adequacy criteria

Elements Vs. Paths

- Statement, Branch, Condition Coverage all focus on *one element* at a time.
- A test executes a ***path***, not a single element.
- Each element on that path is dependent on the others.



Elements Vs. Paths

- There are different control paths through a program…
- … And different ways that data passed along paths can influence execution.
- Important to examine not just elements, but paths.

```
boolean A = …
boolean B = …
boolean expr = A || B;
```

Fault in definition

Corrupts definition of expr if B = False

```
if (expr && C) {
    System.out.println(“Here I am!”);
}
```

expr can corrupt outcome if C = True

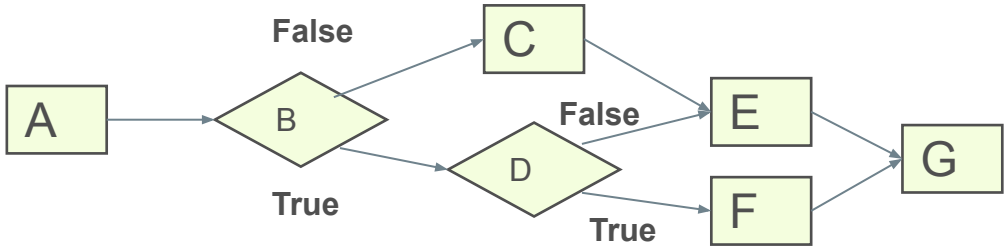
Today's Goals

- Introduce Path Coverage
- Data Flow Coverage Criteria
  - Focus on how information spreads through a program.
  - Based on Definition-Use Pairs
    - (Where is X defined? Where is each definition of X used?)

Path Coverage

Path Coverage

- Path coverage requires that all paths through the CFG are covered.

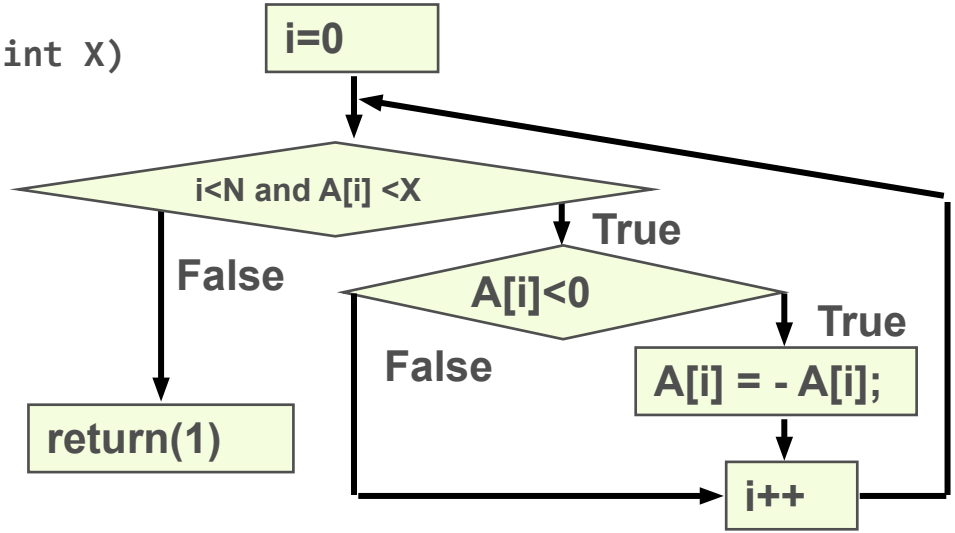


Paths:
A, B, C, E, G
A, B, D, E, G
A, B, D, F, G

- Coverage = $\frac{\text{Number of Paths Covered}}{\text{Number of Total Paths}}$

Path Coverage

```
public int flipSome(int[] A, int N, int X)
{
     int i=0;
     while (i<N and A[i] <X)
     {
          if (A[i]<0)
               A[i] = - A[i];
          i++;
     }
     return A;
}
```

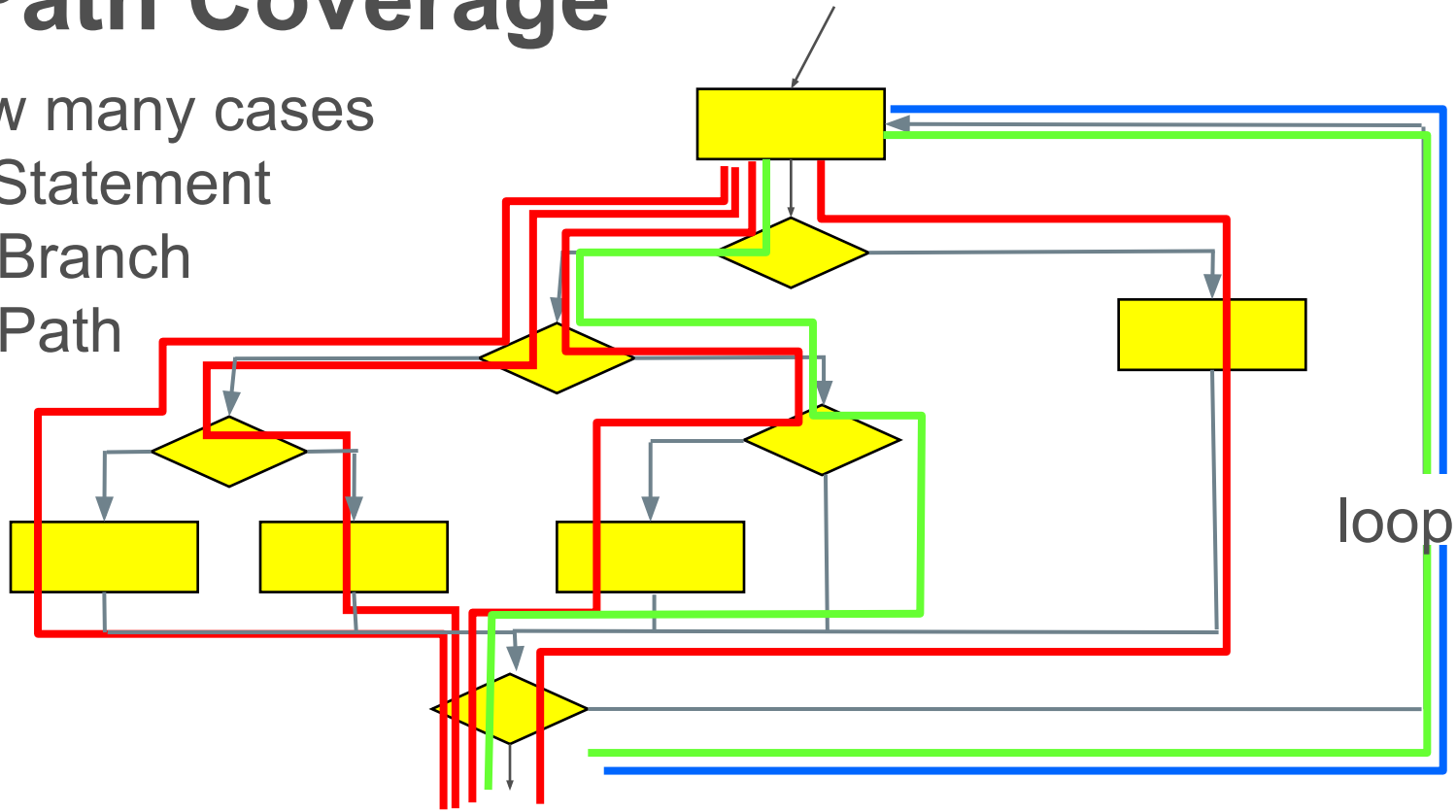


Path coverage is a powerful coverage metric, but is often **impractical**.

- How many paths does this have?
- Each loop cycle is a separate path!

Path Coverage

How many cases
for Statement
Branch
Path



Path coverage with (loop <= 20) requires:
3,656,158,440,062,976 test cases

If you run 1000 tests per second, this will take **116,000 years**.

However, there are ways to get some of the benefits of path coverage without the cost...

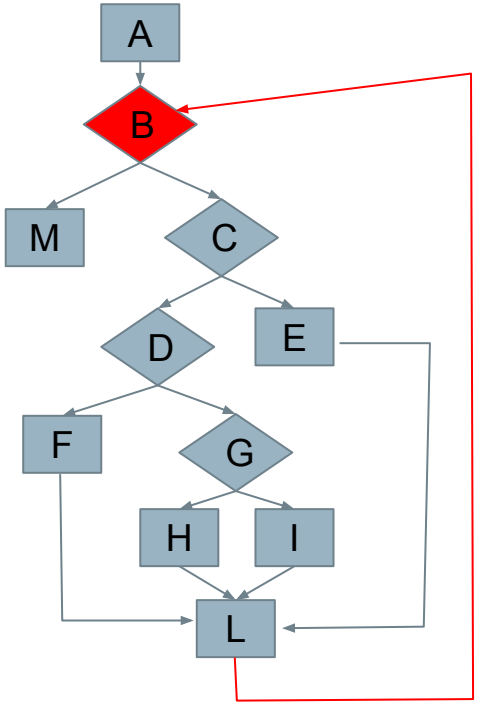
Path Coverage

- Theoretically, a very strong coverage metric.
  - Many faults emerge through sequences of interactions.
- But… Generally impossible to achieve.
  - Loops result in an infinite number of path variations.
  - Even ignoring loops, many paths through code.

Boundary Interior Coverage

- Groups paths that differ only in the subpath they follow when repeating the body of a loop.
  - Executing loop 20 times is different than executing it twice, but same **subpaths** repeat over and over.
  - **Unroll** loop in CFG into distinct subpaths, and cover those instead of worrying about loop cycles.

Boundary Interior Coverage



A -> B -> M
A -> B -> C -> E -> L -> B
A -> B -> C -> D -> F -> L -> B
A -> B -> C -> D -> G -> H -> L -> B
A -> B -> C -> D -> G -> I -> L -> B

Boundary Interior Coverage

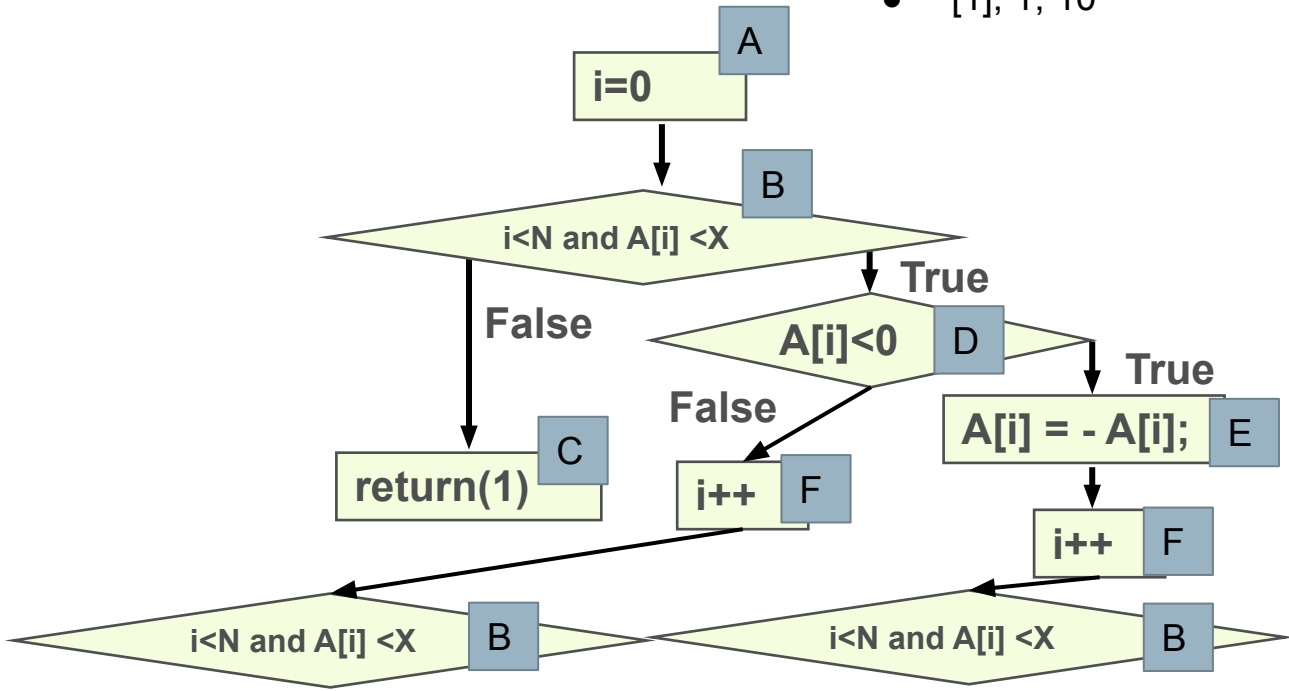
```
public int flipSome(int[] A, int N, int X)
{
     int i=0;
     while (i<N and A[i] <X)
     {
          if (A[i]<0)
               A[i] = - A[i];
          i++;
     }
     return A;
}
```

Test Input

- [ ], 0, 10
- [-1], 1, 10
- [1], 1, 10

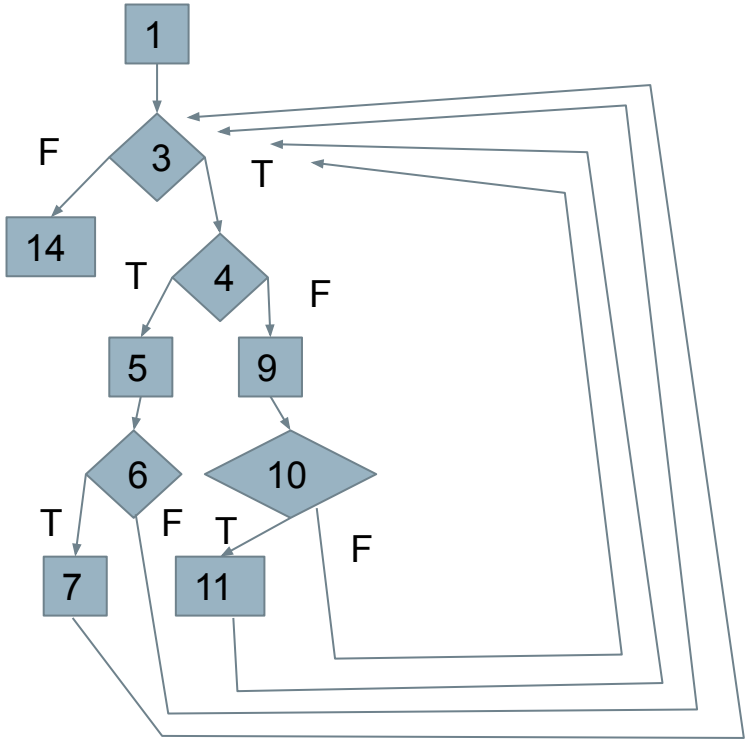
Paths:

- A, B, C
- A, B, D, F, B
- A, B, D, E, F, B



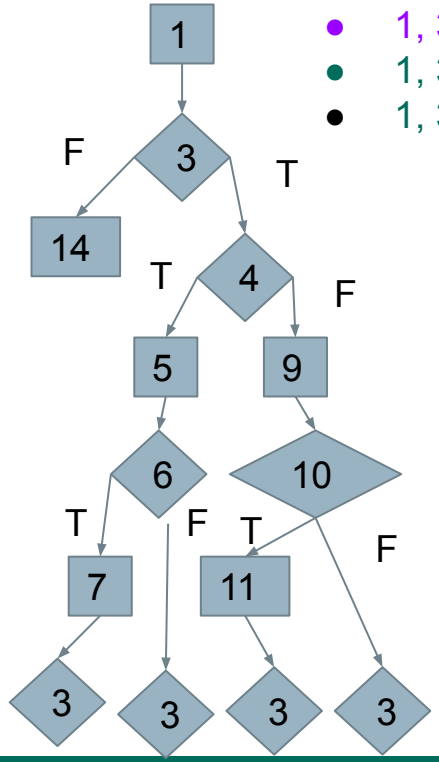
Boundary Interior Example

```
1. public int doSomething(int x, int y)
2. {
3.    while(y > 0) {
4.          if(x > 0) {
5.                y = y - x;
6.                if (y > 0)
7.                    System.out.println("Y: " + y);
8.          }else {
9.                x = x + 1;
10.               if (x <= 0)
11.                   System.out.println(X: " + x);
12.         }
13.   }
14.   return x + y;
15. }
```



Boundary Interior Example

```
1. public int doSomething(int x, int y)
2. {
3.    while(y > 0) {
4.          if(x > 0) {
5.                y = y - x;
6.                if (y > 0)
7.                    System.out.println("Y: " + y);
8.          }else {
9.                x = x + 1;
10.               if (x <= 0)
11.                   System.out.println(X: " + x);
12.         }
13.   }
14.   return x + y;
15. }
```



Paths:

- 1, 3-F, 14
- 1, 3-T, 4-T, 5, 6-T, 7, 3
- 1, 3-T, 4-T, 5, 6-F, 3
- 1, 3-T, 4-F, 9, 10-T, 11, 3
- 1, 3-T,4-F, 9, 10-F, 3

Test Input:

- 10, -1
- 3, 4
- -1, 1

Number of Paths

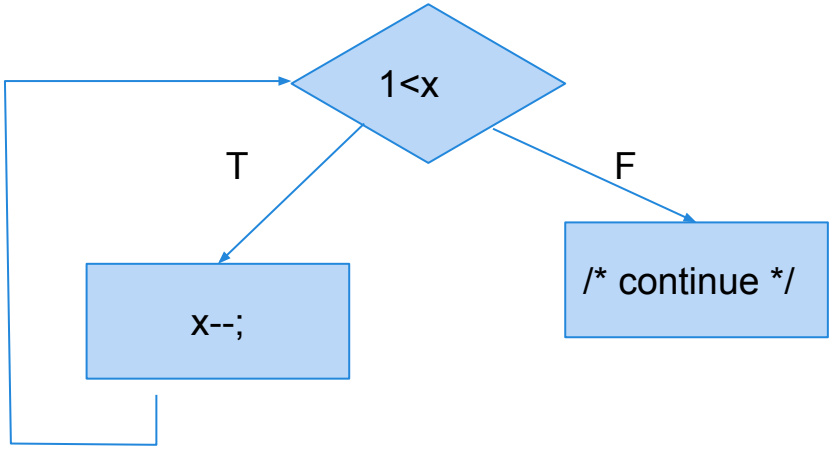
- Boundary Interior Coverage bounds number of paths.
  - However, still exponential.
    - N non-loop branches results in 2^N paths.
- Additional limitations may need to be imposed.

```
if (a)      S1;
if (b)      S2;
if (c)      S3;
…
if (x)      SN;
```

Data Flow

Control Flow

- Capture how execution navigates between blocks of statements.
- We care about a statement's effect **only when it affects the path**.
  - Deemphasizes information being transmitted.



Data Flow

- Program statements compute and transform data…
- Reason about data dependence
  - A variable is used here.
    - Where does its value come from?
  - Is this value ever used?
  - Is this variable properly initialized?
  - If the expression assigned to a variable is changed what else would be affected?

Data Flow

- Basis of compiler optimization.
- Used to derive test cases.
  - Have we covered the dependencies?
- Used to detect faults and other anomalies.
  - When can we cache result of a calculation instead of recalculating it?
  - Can we eliminate a variable definition?

Definition-Use Pairs

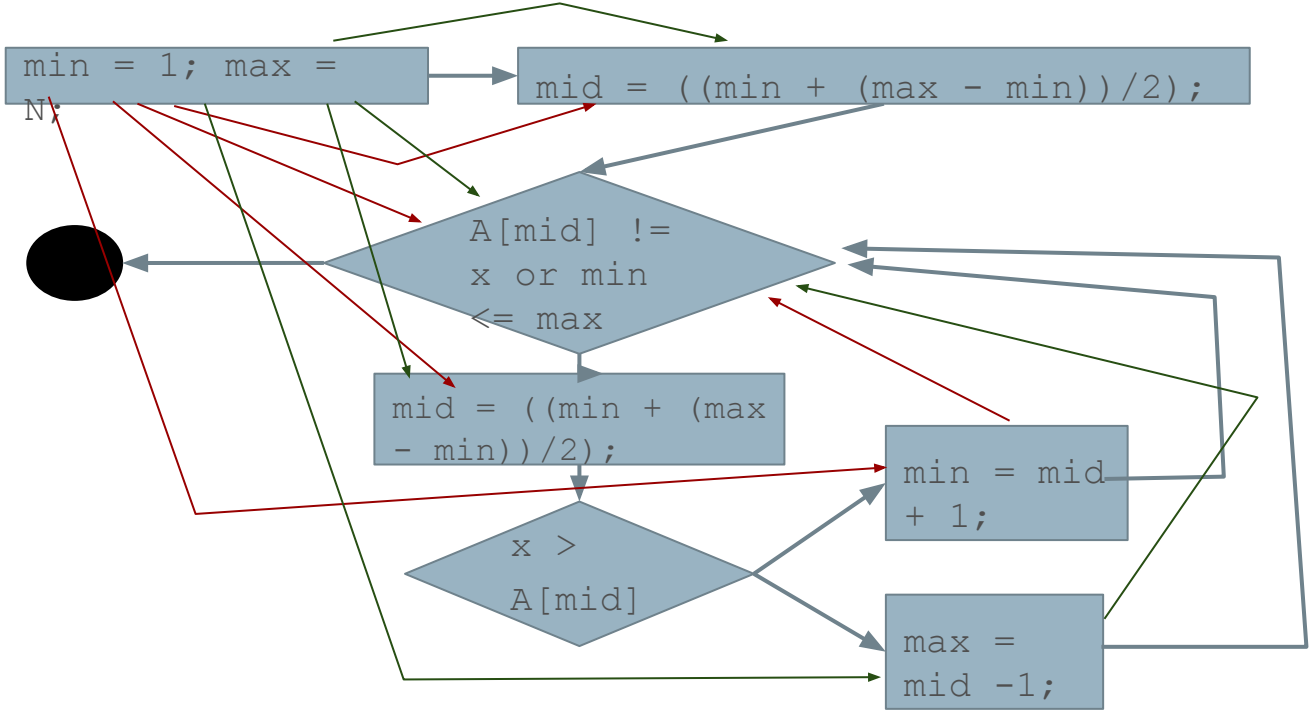
- Data is defined.
  - … and data is used.
- **Pairs of definitions and uses** capture data flow.
  - **Definitions** - when variables are declared, initialized, assigned values, or received as parameters.
  - **Uses** - when variables referenced in expressions, parameter passing, return statements.

Definitions and Uses

```
1.  min = 1;
2.  max = N;
3.  mid = ((min + (max - min))/2);
4.  while (A[mid] != x or min <= max){
5.      mid = ((min + (max - min))/2);
6.      if (x > A[mid]){
7.          min = mid + 1
8.      } else {
9.          max = mid - 1;
10.     }
11. }
```

1. **def** - min
2. **def** - max, **use** - N
3. **def** - mid, **use** - min, max
4. **use** - A[mid], mid, x, min, max
5. **def** - mid, **use** - min, max
6. **use -** x, A[mid], mid
7. **def -** min, **use** - mid
8. -
9. **def -** max, **use -** mid

Definitions and Uses



1. **def** - min
2. **def** - max, **use** - N
3. **def** - mid, **use** - min, max
4. **use** - A[mid], mid, x, min, max
5. **def** - mid, **use** - min, max
6. **use -** x, A[mid], mid
7. **def -** min, **use** - mid
8. -
9. **def -** max, **use -** mid

Definition-Use (DU) Pairs

- We can say there is a **DU pair** when:
  - There is a **definition** of variable X at location A.
  - Variable X is **used** at location B.
  - A control-flow **path** exists from A to B.
  - and the path is **definition-clear** for X from A to B.
- If X is redefined, original definition is **killed** and pair is now between new definition and use in B.

Example - Definition-Use Pairs

```
1.  min = 1;
2.  max = N;
3.  mid = ((min + (max - min))/2);
4.  while (A[mid] != x or min <= max){
5.      mid = ((min + (max - min))/2);
6.      if (x > A[mid]){
7.          min = mid + 1
8.      } else {
9.          max = mid - 1;
10.     }
11. }
```

DU Pairs

min: (1, 3), (1, 4), (1, 5), (7, 4), (7, 5)
max: (2, 3), (2, 4), (2, 5), (9, 4), (9, 5)
N: (0, 2)
mid: (3, 4), (5, 6), (5, 7), (5, 9), (5, 4)
x: (0, 4), (0, 6)
A: (0, 4), (0, 6)

Example - GCD

```
1.  public int gcd(int x, int y){
2.      int tmp;
3.      while(y!=0){
4.          tmp = x % y;
5.          x = y;
6.          y = tmp;
7.      }
8.      return x;
9.  }
```

1. def: x, y
2. def: tmp
3. use: y
4. use: x, y
   def: tmp
5. use: y
   def: x
6. use: tmp
   def: y
7. -
8. use: x

Example - GCD

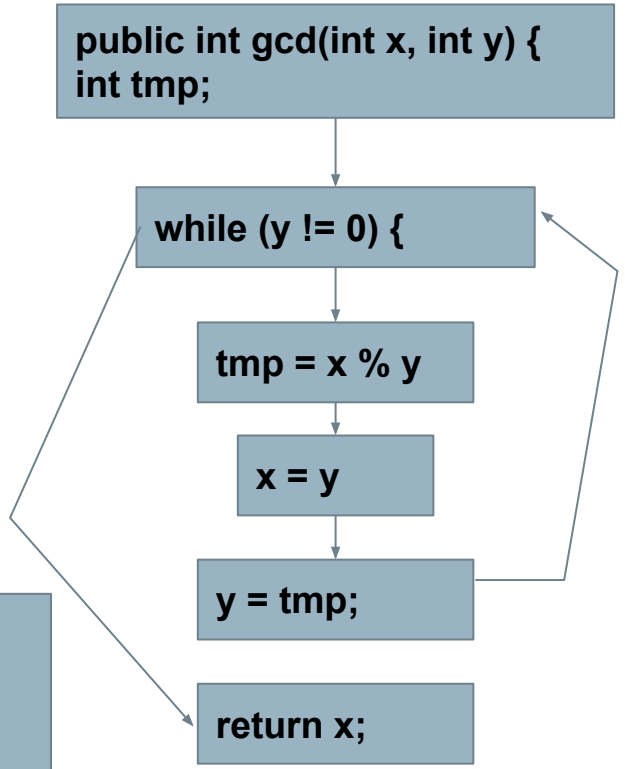
```
1.  public int gcd(int x, int y){
2.      int tmp;
3.      while(y!=0){
4.          tmp = x % y;
5.          x = y;
6.          y = tmp;
7.      }
8.      return x;
9.  }
```

Def-Use Pairs

x: (1, 4), (5, 4), (5, 8), (1, 8)

y: (1, 3), (1, 4), (1, 5), (6, 3), (6, 4), (6, 5)

tmp: (4, 6)



Example - collapseNewlines

```
7. public static String collapseNewlines(String argStr)
8. {
9.    char last = argStr.charAt(0);
10.   StringBuffer argBuf = new StringBuffer();
11.
12.   for(int cIdx = 0; cIdx < argStr.length(); cIdx++)
13.   {
14.         char ch = argStr.charAt(cIdx);
15.         if(ch != '\n' || last != '\n')
16.         {
17.               argBuf.append(ch);
18.               last = ch;
19.         }
20.   }
21.
22.   return argBuf.toString();
23. }
```

Variable	D-U Pairs
argStr	(7, 9), (7,12), (7, 14)
last	(9, 15), (18, 15)
argBuf	(10,22), (17, 22)
cIdx	(12, 12), (12, 14)
ch	(14, 15), (14, 17), (14, 18)

Let’s Take a Break

Dealing With Arrays and Pointers

- Arrays and pointers (including object references and arguments) introduce issues.
  - It is not possible to determine whether two access refer to the same storage location.
    - `a[x] = 13;`
      `k = a[y];`
      - Are these a def-use pair?
    - `a[2] = 42;`
      `i = b[2];`
      - Are these a def-use pair?

Aliasing

- Two names refer to the same memory location.
  - ```
    int[] a = new int[3];
    int[] b = a;
    a[2] = 42;
    i = b[2];
    ```
- Worse in C:

```
p = &b;
*(p + i) = k;
```
```

Uncertainty

- Aliasing introduces uncertainty.
  - Instead of definition or use of one variable, may have a potential def or use of a set of variables.
- Safest: treat **any** use of a potential alias of V as a use of V.
  - Creates many def-use pairs (some may not be real), but avoids missing pairs.

Dealing With Uncertainty

- Can treat all potential aliases as definitions and uses:

```
a[1] = 13;
k = a[2];
```

No uncertainty. Def of **a[1]**, use of **a[2]**.

```
a[x] = 13;
k = a[y];
```

Potential uncertainty. Treat as def and use of **array a**.

- Can be **very imprecise**.
  - They are only really the same if x and y are the same.

Dealing With Uncertainty

- Option 2: Treat uncertainty about aliases like uncertainty about control flow.

```
a[x] = 13;
k = a[y];
```

```
a[x] = 13;
if(x == y)    k = a[x];
else          k = a[y];
```

- Rewrite code to make references explicit.
- In transformed code, all array references are distinct.

Situational Def-Use Pairs

- `++counter, counter++, counter+=1`
  `counter = counter + 1`
  - Use of `counter` then a new definition.
- `char *ptr = *otherPtr`
  - Definition of string `*ptr`
  - Use of memory index `ptr`, string `*otherPtr`, and memory index `otherPtr`.
  - `ptr++`
    - Use of memory index ptr, definition of both memory index and string *ptr (change to index moves pointer to a new location).

Data Flow Coverage Criteria

Overcoming Limitations of Path Coverage

- We can potentially expose many faults by targeting particular paths of execution.
- What are the **important paths** to cover?
  - Some methods impose heuristic limitations.
  - Use data flow to select paths based on how one element can affect the computation of another.

Choosing the Paths

- Computing the wrong value leads to a failure **only when that value is used**.
  - Ensure that definitions are actually used by covering paths from definitions to uses.
  - All DU Pair Coverage, All DU Paths Coverage, All Definitions Coverage
    - Varying power and cost.

All DU Pair Coverage

- Requires each DU pair be exercised in at least one program execution.
  - Counts if we cover **any of the paths** between a definition and its use.
  - Can easily achieve structural coverage without covering all DU pairs.
- Coverage = $\frac{\text{number exercised DU pairs}}{\text{number of DU pairs}}$

All DU Pairs Coverage Example

```
1. public int doSomething(int x, int y)
2. {
3.    while(y > 0) {
4.          if(x > 0) {
5.                y = y - x;
6.                if (y > 0)
7.                    System.out.println("Y: " + y);
8.          }else {
9.                x = x + 1;
10.               if (x <= 0)
11.                   System.out.println(X: " + x);
12.         }
13.   }
14.   return x + y;
15. }
```

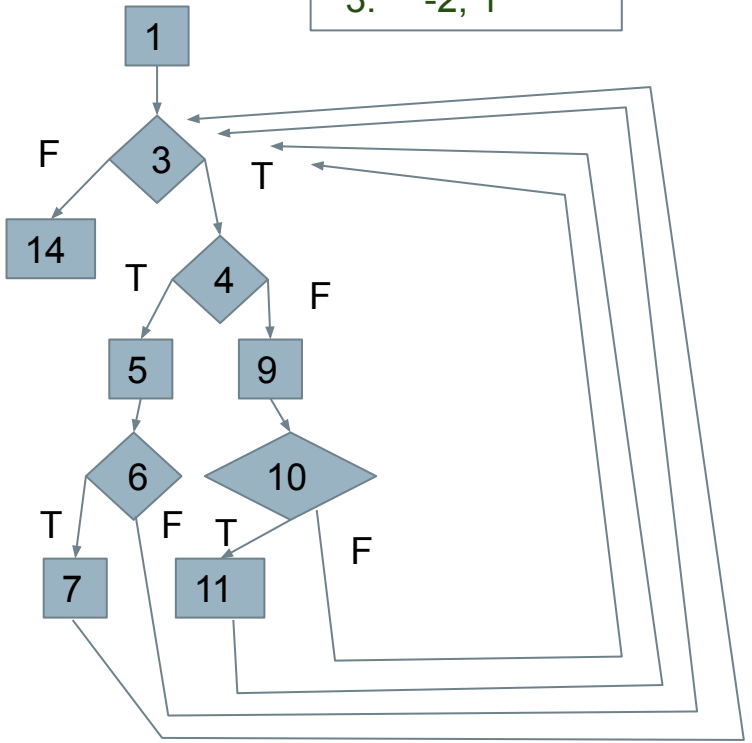
X:
(1, 4), (1, 5), (1, 9), (1, 14)
(9, 10), (9, 11), (9, 4), (9, 5), (9, 9), (9, 14)

Y:
(1, 3), (1, 5), (1, 14)
(5, 6), (5, 7), (5, 3), (5, 5), (5, 14)

X: (1, 4), (1, 5), (1, 9), (1, 14), (9, 10), (9, 11), (9, 5), (9, 9), (9, 14)
Y: (1, 3), (1, 5), (1, 14), (5, 6), (5, 7), (5, 3), (5, 5), (5, 14)

Test Input:
1. -1, 1
2. 3, 7
3. -2, 1

```
1. public int doSomething(int x, int y)
2. {
3.    while(y > 0) {
4.          if(x > 0) {
5.                y = y - x;
6.                if (y > 0)
7.                    System.out.println("Y: " + y);
8.          }else {
9.                x = x + 1;
10.               if (x <= 0)
11.                   System.out.println(X: " + x);
12.         }
13.   }
14.   return x + y;
15. }
```



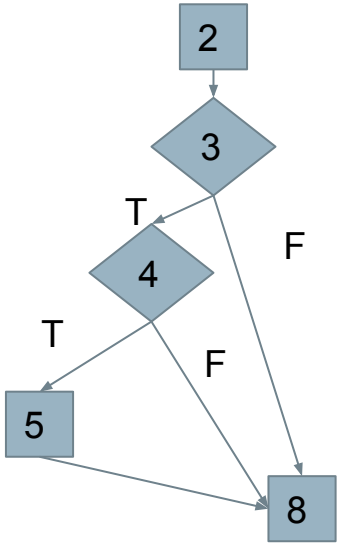
All DU Paths Coverage

- A use may be reachable along several paths from the definition.
  - Cover **all non-looping paths** for each DU pair.
  - Can reveal faults where a path is exercised that should use a certain definition but doesn't.

$$\text{Coverage} = \frac{\text{number of exercised DU paths}}{\text{number of DU paths}}$$

All DU Paths Example

```
1.  ...
2.  int x = 1;
3.  if (y > 7) {
4.      if (z > 5) {
5.          z = x + 5;
6.      }
7.  }
8.  y = x + 7;
9.  ...
```



DU Pair (2, 8) for X can be reached along multiple paths.

- 2, 3T, 4T, 5, 8
- 2, 3T, 4F, 8
- 2, 3F, 8

Test Input:

- y = 10, z = 6
- y = 10, z = 3
- y = 2, z = (anything)

Path Explosion Problem

- Even without looping paths, number of DU paths can be exponential.
  - Code between definition and use can be irrelevant to that variable, but contains many paths.

```
public void countBits(char ch){
     int count = 0;
     if (ch & 1)   ++count;
     if (ch & 2)   ++count;
     if (ch & 4)   ++count;
     if (ch & 8)   ++count;
     if (ch & 16)  ++count;
     if (ch & 32)  ++count;
     if (ch & 64)  ++count;
     if (ch & 128) ++count;
     System.out.println(ch + " has " +
count + "bits set to 1");
}
```

All Definitions Coverage

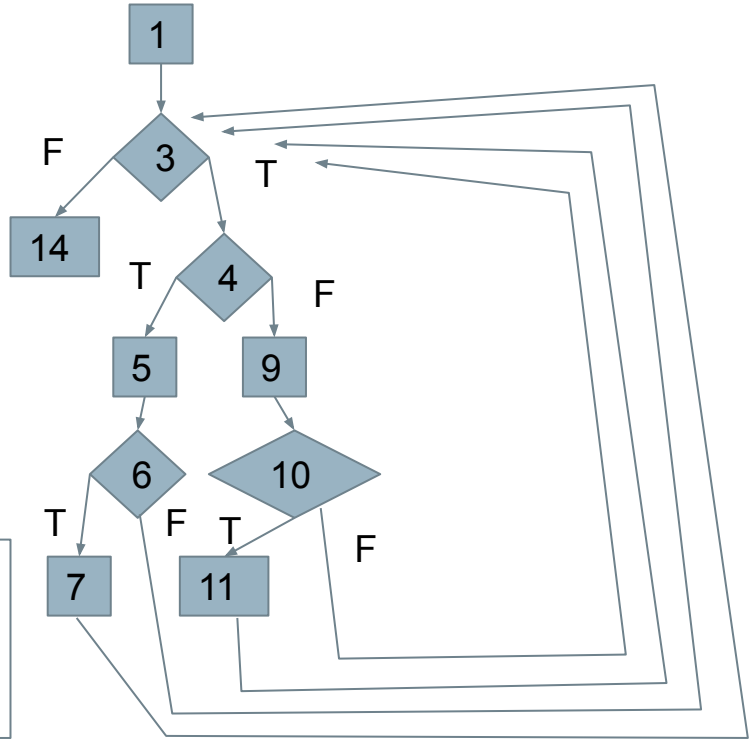
- Alternative when others are too expensive.
  - **Pair each definition with at least one use.**
  - Skips many DU pairs, but ensures each definition tried.

$$\text{Coverage} = \frac{\text{number of covered definitions}}{\text{number of definitions}}$$

X: (1, 4), (1, 5), (1, 9), (1, 14), (9, 10), (9, 11), (9, 5), (9, 9), (9, 14)
Y: (1, 3), (1, 5), (1, 14), (5, 6), (5, 7), (5, 3), (5, 5), (5, 14)

X: Definitions on lines 1, 9
Y: Definitions on lines 1, 5

```
1. public int doSomething(int x, int y)
2. {
3.    while(y > 0) {
4.         if(x > 0) {
5.              y = y - x;
6.              if (y > 0)
7.                   System.out.println("Y: " + y);
8.         }else {
9.              x = x + 1;
10.             if (x <= 0)
11.                  System.out.println(X: " + x);
12.        }
13.   }
14.   return x + y;
15. }
```



- Any input covers (1, -) pairs.
- Reaching lines 5, 9 covers (5,14) and (9,14) pairs.

Infeasibility Problem

- Metrics may ask for impossible test cases.
- Path-based metrics may require infeasible combinations of feasible elements.
  - Alias analysis may add additional infeasible paths.
- All Definitions, All DU-Pairs Coverage reasonable.
  - All DU-Paths is much harder!

Activity - DU Pair Coverage

- Identify all DU pair
- Write **your own** test input to achieve All DU Pair Coverage.
  - e.g., Input (1, 2)
    For x, covers pairs:
    (1,4), …

```
1. int doSomething(int x, int y)
2. {
3.       while(y > 0) {
4.           if(x > 0) {
5.               y = y - x;
6.           }else {
7.               x = x + 1;
8.           }
9.       }
10.      return x + y;
11. }
```

Activity - DU Pairs

```
1. int doSomething(int x, int y)
2. {
3.       while(y > 0) {
4.           if(x > 0) {
5.               y = y - x;
6.           }else {
7.               x = x + 1;
8.           }
9.       }
10.      return x + y;
11. }
```

Variable	Defs	Uses
x	1, 7	4, 5, 7, 10
y	1, 5	3, 5, 10

Variable	D-U Pairs
x	(1, 4), (1, 5), (1, 7), (1, 10), (7, 4), (7, 5), (7, 7), (7, 10)
y	(1, 3), (1, 5), (1, 10), (5, 3), (5, 5), (5, 10)

Activity - DU Pairs

```
1. int doSomething(int x, int y)
2. {
3.       while(y > 0) {
4.           if(x > 0) {
5.               y = y - x;
6.           }else {
7.               x = x + 1;
8.           }
9.       }
10.      return x + y;
11. }
```

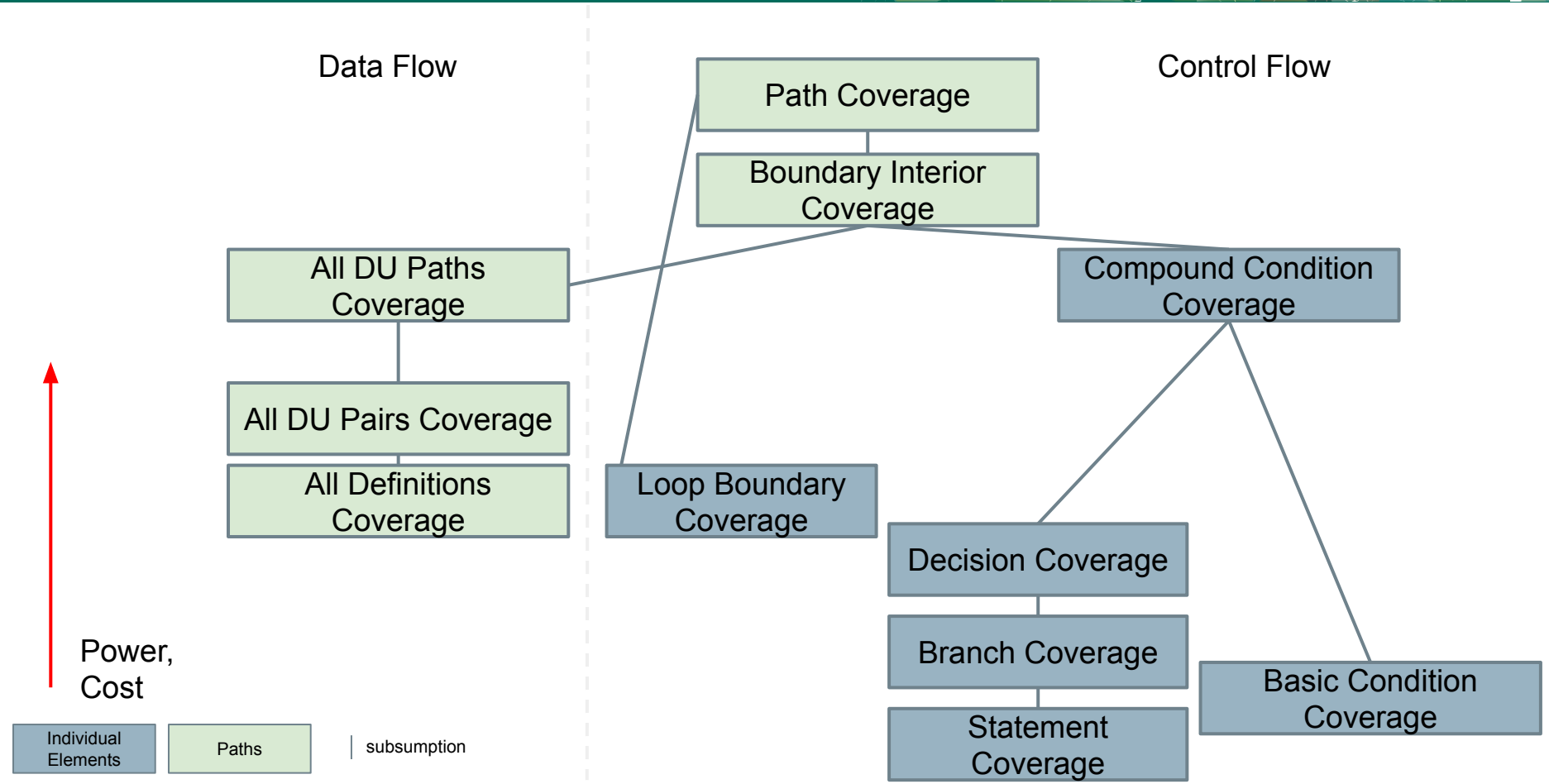
Variable	Defs	Uses
x	1, 7	4, 5, 7, 10
y	1, 5	3, 5, 10

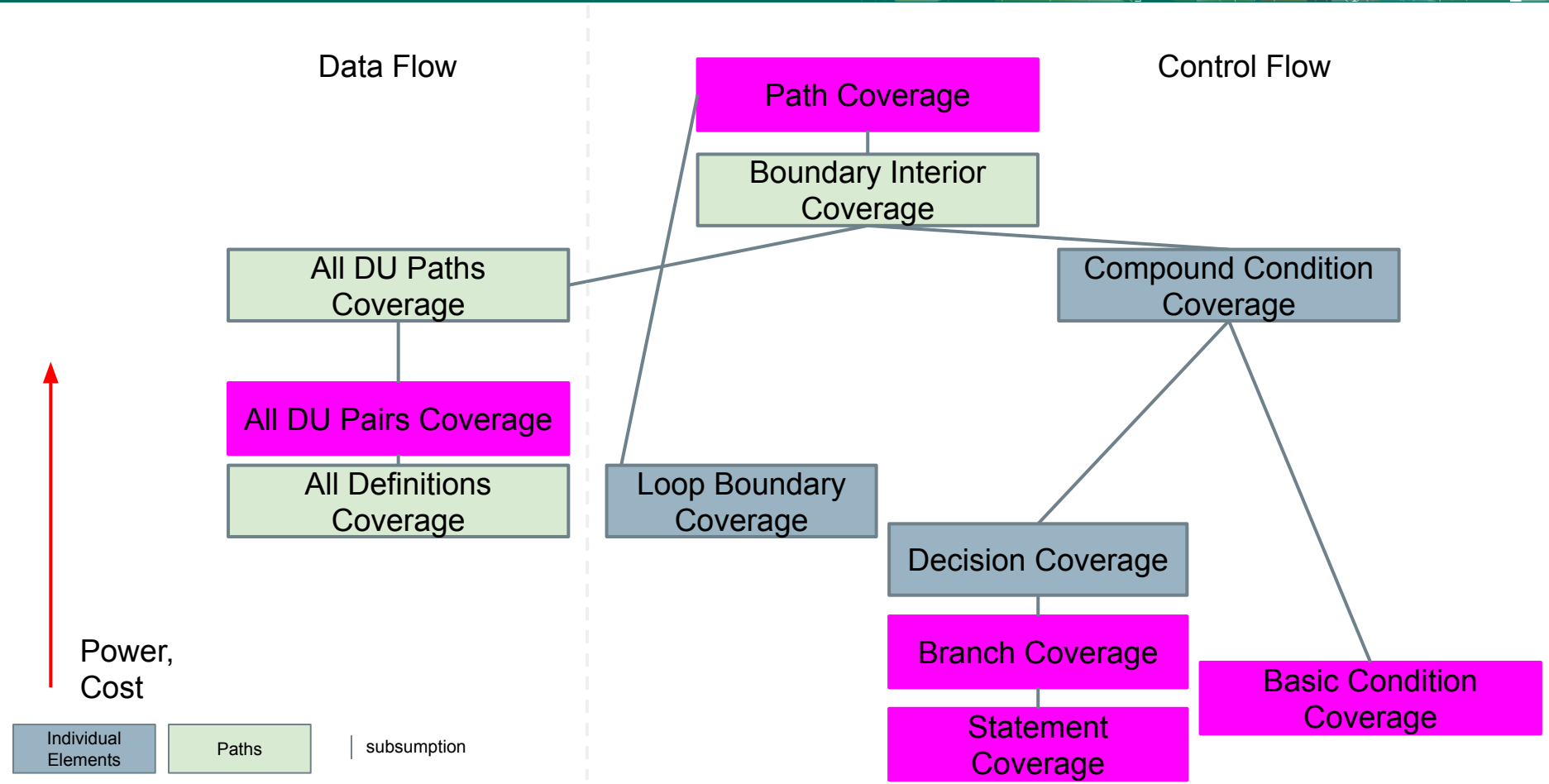
Variable	D-U Pairs
x	~~(1, 4), (1, 5), (1, 7), (1, 10), (7, 4), (7, 5), (7, 7), (7, 10)~~
y	~~(1, 3), (1, 5), (1, 10), (5, 3), (5, 5), (5, 10)~~

Test Input 1: (x = 1, y = 2)
Covers lines 1, 3, 4, 5, 3, 4, 5, 3, 10

Test Input 2: (x = -1, y = 1)
Covers lines 1, 3, 4, 6, 7, 3, 4, 6, 7, 3, 4, 5, 3, 10

Test Input 3: (x = 1, y = 0)
Covers lines 1, 3, 8


Data Flow
Control Flow
Path Coverage
Boundary Interior Coverage
All DU Paths Coverage
Compound Condition Coverage
All DU Pairs Coverage
All Definitions Coverage
Loop Boundary Coverage
Decision Coverage
Branch Coverage
Basic Condition Coverage
Statement Coverage
Power, Cost
Individual Elements
Paths
subsumption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We Have Learned

- Control-flow and data-flow capture important paths in program execution.
- Analysis of how variables are defined and then used and the dependencies between definitions and usages can help us reveal important faults.
- Many forms of analysis can be performed using data flow information.

We Have Learned

- If there is a fault in a computation, we can observe it by looking at where the computation is used.
- By identifying DU pairs and paths, we can create tests that trigger faults along those paths.
  - All DU Pairs coverage
  - All DU Paths coverage
  - All Definitions coverage

Next Time

- Next lecture - Mutation Testing

- Assignment 3
  - Due March 1!
  - We have covered everything on it.

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